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Nonholonomic Mechanics and Control

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engineering from the University of California, Berkeley, in 1989 and 1992, respectively. After receiving his Ph.D., he was a postdoctoral fellow at the Ecole des Mines de Paris in Fontainebleau, France. In September 1992, he joined the faculty of the Electrical Engineering Department at the University of Minnesota, where he was an assistant professor until September 1997. In 1997, he joined the faculty of the Electrical and Computer Engineering Department at the University of California, Santa Barbara, where he is currently a professor and director of the Center for Control Engineering and Computation. He has received the National Science Foundation Research Initiation and CAREER Awards, the 1998 IEEE Leon K. Kirchmayer Prize Paper Award, and the 1998 George S. Axelby Outstanding Paper Award. He was the recipient of the first SIAM Control and Systems Theory Prize in 1998. He was also the recipient of the 1999 Donald P. Eckman Award and the 2001 O. Hugo Schuck Best Paper Award, both given by the American Automatic Control Council. He is a Fellow of the IEEE.

Luca Zaccarian graduated with a degree in electronic engineering from the University of Roma Tor Vergata in 1995, where he also received his Ph.D. degree in computer science and control engineering in 2000. In 1998–2000, he spent two years as a visiting researcher at the Center for Control Engineering and Computation of the University of California, Santa Barbara. During the summers of 2003 and 2004, he was a visiting professor at the Electrical and Electronic Engineering Department of the University of Melbourne (Australia). Since 2000, he has been an assistant professor (“ricercatore”) in control engineering at the University of Roma Tor Vergata. He was the recipient of the 2001 O. Hugo Schuck Best Paper Award given by the American Automatic Control Council.

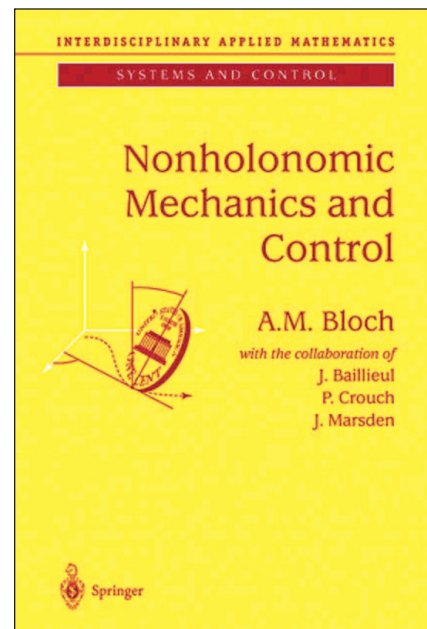
Nonholonomic Mechanics and Control by A.M. Bloch with the collaboration of J. Baillieul, P. Crouch, and J. Marsden. Springer, 2003. ISBN 0-387-95535-6, US\$69.95. Reviewed by A.J. van der Schaft.

Holonomic and Nonholonomic Constraints

As aptly formulated in its preface, *Nonholonomic Mechanics and Control* links control theory with a geometric view of classical mechanics in both its Lagrangian and Hamiltonian formulations and, in particular, with the theory of mechanical systems subject to kinematic motion constraints. The analysis and control of mechanical systems has been an active research area over the last several decades. The book aims to present some of this material, often scattered throughout the literature, in a cohesive manner.

Kinematic constraints are classically divided into two classes: constraints that can be integrated to yield constraints on the position variables, called *holonomic* constraints, and constraints for which this integration is not possible, called *nonholonomic* constraints. A typical example of a nonholonomic constraint is a wheel rolling vertically *without slipping* on a surface. The constraint on the allowable velocity (the point of contact of the wheel with the surface cannot slip in all directions) cannot be integrated to yield a constraint on the *position* of the wheel. This nonintegrability is intuitively clear, as illustrated by the fact that an automobile can go anywhere it pleases by suitable maneuvering. (This example hints at the intimate relationship between nonholonomic constraints and controllability.) Loosely speaking, mechanical systems with holonomic constraints can be reduced to lower dimensional mechanical systems without constraints; for systems with nonholonomic constraints, this reduction is not possible, and as a result some distinguishing features arise.

Systems with nonholonomic constraints are not only well motivated by applications such as mobile robots but are intellectually interesting as well. From an analytical and geometric point of view, nonholonomic constraints present a class of systems that cannot be described in the standard Lagrangian or Hamiltonian framework; the powerful machinery developed for the standard case must be modified and extended. At the same time, nonholonomic constraints may actually *help* in controlling a system. In particular, for *underactuated* mechanical systems, the presence of nonholonomic constraints in directions that are not directly actuated is helpful for controllability. Interestingly enough, the geometric/analytical and nonlinear control points of view appear to be intimately linked, giving rise to a rich theory that can be illustrated on simple mechanical examples. It is reasonably “easy” to write a difficult and obscure book about this theory, but it is much more difficult to write a book that provides clear access to the topic, which intrinsically has many facets. The author has invested considerable effort in achieving this goal.



Contents

The text begins with four introductory chapters. After introducing Hamilton's principle and the Lagrange-d'Alembert principle, Chapter 1 presents a collection of mechanical systems subject to nonholonomic kinematic constraints. The treatment of these examples illustrates some of the ideas and phenomena discussed in detail in the later chapters. As such, this chapter gives the basic flavor of the material and provides an ideal introduction to the book. Chapters 2, 3, and 4, respectively titled "Mathematical Preliminaries," "Basic Concepts in Geometric Mechanics," and "Introduction to Aspects of Geometric Control Theory," give a brief overview of the mathematical background needed for the rest of the book and introduce key definitions and notation. These chapters can be skipped by readers who are acquainted with this material and can be returned to as needed.

Although, unavoidably, the opening chapters provide only a "crash course" at some points, the material has been written with much care. In fact, in many cases, the clarity of the presentation is unmatched elsewhere in the literature. An example is the treatment of Ehresmann connections in Section 2.9, where the geometric definition and interpretation is linked to the algebraic definition in a very insightful way. Also, the relation with linear (Koszul) connections on tangent bundles is clearly explained. (I would have also linked the definition of a linear connection on a tangent bundle to the geometric interpretation of an Ehresmann connection.) As a result, these introductory chapters provide stimulating reading for a reader well versed in this material as well as for the novice.

A core chapter of the text is Chapter 5, which deals with the geometric formulation of mechanical systems subject to nonholonomic kinematic constraints, both in the Lagrangian and Hamiltonian frameworks. This chapter provides a nice synthesis of the litera-

ture, some of which is quite recent. Chapter 6 deals with the control of nonholonomic systems, and the control of mechanical systems subject to nonholonomic constraints. The terminology *nonholonomic (control) systems*, which may be somewhat confusing, originates from the kinematic model of a mechanical system subject to nonholonomic kinematic constraints, for which the feasible space of velocities on the configuration space of the mechanical system is represented as the span of input vector fields. Pure nonholonomicity of the kinematic constraints (in the sense that no part of them can be integrated to yield constraints on the configuration variables) then corresponds to *controllability* of the ensuing control system. Controllability can be characterized by a full-rank condition on the space of vector fields generated by taking (possibly repeated) Lie brackets of the input vector fields. Thus, nonholonomicity is intimately linked to controllability. On the other hand, the linear control paradigm of equivalence between controllability and pole placement breaks down in this case in the sense that there does *not* exist an asymptotically stabilizing continuous state feedback law. The second part of Chapter 6 deals with the control of dynamical models of mechanical systems subject to kinematic constraints and with external input forces.

Chapter 7 describes another interesting connection between nonlinear control theory and nonholonomic constraints, namely, the variational formulation of mechanical systems subject to nonholonomic constraints, as in Hamilton's principle as opposed to the Lagrange-d'Alembert principle. The difference is that, in the first case, the time-integral of the Lagrangian is extremized over all configuration trajectories whose tangent vectors satisfy the kinematic constraints, while in the latter case the conditions for extremality of an arbitrary curve in the configuration space is checked with respect to all tangent vectors (classically called vir-

tual displacements) satisfying the kinematic constraints. This subtlety is not easy to grasp and may seem to be a purely academic discussion (in the bad sense of the word). However, it turns out that the equations of motion obtained from Hamilton's principle are generally different from the "correct" equations of motion obtained by applying the Lagrange-d'Alembert principle; in fact, these equations are the same if and only if the constraints are holonomic!

On the other hand, the framework of Hamilton's principle is linked to nonlinear optimal control theory, while the special case of optimal control of nonholonomic control systems leads to the interesting mathematical topic of sub-Riemannian geometry. Chapter 8 deals with the key problem of stability of mechanical systems with nonholonomic constraints and shows how the delicate stability theory of ordinary Hamiltonian systems can be extended to this case. Finally, Chapter 9 provides a control counterpart by studying the stabilization or setpoint regulation of underactuated mechanical systems. The main method is the theory of controlled Lagrangians. (The inclusion of gyroscopic forces in the Lagrangian description is equivalent to interconnection-damping assignment control of port-Hamiltonian systems.) This method is based on the use of state feedback to shape another *virtual* mechanical system with energy that has a critical point at the desired setpoint. Another method treated is the theory of averaging for underactuated mechanical systems.

Summary

This book is a welcome addition to the existing literature and will certainly become a standard reference. Related books with a different emphasis and partly complementary choice of topics include [1] and the recent [2]. It is to be expected that Bloch's book will be a continuing source of inspiration for further research in this area. Indeed, many aspects remain to be investigated,

especially from the systems and control point of view. While important basic understanding has been reached for trajectory planning and classical control problems like setpoint stabilization, the general problem of controlling (by state feedback laws or by interconnection with other dynamical systems) the dynamical behavior of the physical system is still largely open. Important issues here are understanding the robust dynamical behavior of systems with nonholonomic constraints and the exploration of new control paradigms.

In summary, this is a delightful book that will be valuable for both the control community and researchers working on the geometric theory of mechanical systems. With its extensive illustrations and exercises, this book is eminently suited for a graduate course. The author should be congratulated for such an admirable job.

References

- [1] J. Cortés Monforte, *Geometric, Control and Numerical Aspects of Nonholonomic Systems* (Lecture Notes in Mathematics 1793). Berlin: Springer-Verlag, 2002.
- [2] F. Bullo and A.D. Lewis, *Geometric Control of Mechanical Systems. Modeling, Analysis, and Design for Simple Mechanical Control Systems* (Texts in Applied Mathematics, 49). New York: Springer-Verlag, 2005.

A.J. (Arjan) van der Schaft received his undergraduate and Ph.D. degrees in mathematics from the University of Groningen, The Netherlands, in 1979 and 1983, respectively. In 1982, he joined the Department of Applied Mathematics, University of Twente, Enschede, The Netherlands, where he is presently a full professor in mathematical systems and control theory. His research interests include the mathematical modeling of physical and engineering systems and the control of nonlinear and hybrid systems. He has served as associate editor for *Systems & Control Letters*, *Journal of Nonlinear Science*, *SIAM Journal on Control*, and *IEEE Transactions on Automatic Control*. Currently, he is associate editor for *Systems and Control*

Letters and editor-at-large for the *European Journal of Control*. He is coauthor of *System Theoretic Descriptions of Physical Systems* (1984), *Variational and Hamiltonian Control Systems* (1987, with P.E. Crouch), *Nonlinear Dynamical Control Systems* (1990, with H. Nijmeijer), *L₂-Gain and Passivity Techniques in Nonlinear Control* (2000), and *An Introduction to Hybrid Dynamical Systems* (2000, with J.M. Schumacher). He is a Fellow of the IEEE.

Introduction to Stochastic Search and Optimization by James C. Spall, Wiley, Hoboken, NJ, 2003, ISBN 0-471-33052-3, US\$105.00. Reviewed by G. Yin.

Search and Optimization Methods

James Spall's book provides a survey of random search (called stochastic search by the author) and optimization methods, including stochastic approximation algorithms, evolutionary computation, Monte Carlo simulation, and related statistical methods. Stochastic approximation algorithms include recursive least squares, adaptive algorithms, and reinforcement learning.

One of the original aims of stochastic approximation is to find roots of a continuous function $f(\cdot)$, where either the precise form of f is not known or f is too complicated to compute and only noisy measurements are available. That is, at iteration n (often referred to as time n) and design point x_n (often referred to as state x_n), it is possible to obtain only a noisy measurement $y_n = f(x_n) + \xi_n$, where ξ_n denotes random noise. In 1951, Robbins and Monro [2] introduced the recursive algorithm $x_{n+1} = x_n + a_n y_n$, where a_n is an appropriately chosen step size. Robbins and Monro coined the name *stochastic approximation* for this procedure. Another one of the original aims of stochastic approximation is to determine the minimizer of a real-valued cost function h using only noisy measurements of $h(x_n)$. To find local minimizers of a smooth function

h , one may try to find the roots of the gradient of h . However, the gradient of h is not available when only noisy measurements of h can be used. In 1952, Kiefer and Wolfowitz [3] proposed another recursive algorithm for solving this problem, in which the gradient of h is replaced by its noisy gradient estimate of finite difference type.

Since stochastic approximation methods were first introduced, there has been significant progress in the development of more sophisticated algorithms. Much of this development has originated from, and has been intertwined with, applications in optimization, control theory, economic systems, signal processing, communication theory, learning, pattern classification, neural networks, and related fields. Emerging applications have been found in wireless communications, repeated stochastic games, and financial engineering. For instance, consider a production planning problem with unreliable machines. Under certain conditions, it can be shown that the optimal control is of the threshold type. For multiple machine problems, it is virtually impossible to find closed-form solutions. However, using stochastic approximation methods, the problem can be recast as a parametric optimization problem that can be solved by

